Tutorial 7A
Advanced Graph Theory

9^{th} September 2014

1. Suppose a connected graph G is decomposed into two graphs G_1 and G_2. Prove that G_1 and G_2 must have a common vertex.

2. Given a hypercube Q_r, find the smallest closed walk and largest length cycle such that they cover all r dimensions of Q_r.

3. G is a simple n-vertex graph. Prove that if $\delta(G) \geq \lfloor \frac{n}{2} \rfloor$, then $\kappa'(G) = \delta(G)$.

4. Find a maximum weighted matching of the following bipartite graph. (Rows and columns represent the vertices in the two partite sets respectively, and $X[i, j]$ represents the weight of the edge between node i $(0 \leq i \leq 4)$ in one partite set to node j $(0 \leq j \leq 4)$ in the second partite set). At each step, clearly show the equality subgraph, the matching/vertex cover found, and the updated cover. [15 marks]

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 7 & 2 \\ 1 & 3 & 4 & 4 & 5 \\ 3 & 6 & 2 & 8 & 7 \\ 4 & 1 & 3 & 5 & 4 \end{pmatrix}$$